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Stabilization of a micro Rotor using Pontraygrin optimal controller

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Abstract—**Without Rotating machines, it is almost impossible to comprehend machineries in modern engineering world .There are just too many associated parts attached with such machines . Many of these machines are required to run at very high speeds sometimes much higher than their first critical speeds. Stiffness and damping properties plays a very important role in the dynamic behaviour of these high speed rotors supported on journal bearings .Rotor-bearing systems exhibit a wide variety of phenomena pertaining to its operations which if not properly addressed and rectified may lead to catastrophic failure of the system. One such rotor-bearing phenomenon is the self-acting rotor vibrations induced by the hydrodynamic bearings and popularly known as oil-whirl and oil-whip which have severe repercussion on the functioning of the rotors. Hydrodynamic bearing and integrated model tend to be unstable due to non-conservative nature of hydrodynamic forces and stiffness coefficients. The purpose of this work is to redeem vibrations of hydrodynamic bearings with elegant control strategies. Attenuation of vibration in this work is done by using optimal control and conformal mapping based fractionalcontroller in such a way such that ultimate skeleton of the controller remains very simple to for practical deployment and also it improves the power utility and reduces overall energy cost.**

I. Introduction

Rotor dynamics in modern day turbo machinery problems must be undertaken with absolutely greater depth as because very frequently operations of rotating machinery take place much above the sub-synchronous or critical speed. It is quite obvious that the rotating machines and its role and applications in industry affect directly the basic economic issues and deals very closely with the human life. Therefore, safe operation of this rotating machinery is an absolute necessity and dynamically to stabilize such machines is an absolute must for safe operations in industries.

The dynamical systems subjected to such displacement dependent forces which cannot be easily be derived from slope or gradient of any potential function may become highly unstable. These self-induced instability is very common in tope's and pendulum like system which was studied by Or (1994), Kirillov(2007), Bou-Rabee et al (2008), and Samantarayetal(2008). The shafts may draw energy from motor drives or energy from inertial parts causing instability. Some common examples of such engineering systems could be elastic structures subjected to aerodynamic forces, dissipative forces in couplings,hydrodynamic forces due to bearing supporting the rotor.

II. Dynamics And Instability Analysis of Micro size Rotors

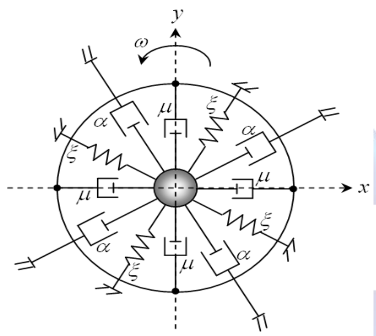


Fig. 1.0 Distributed structure of the internal damping and the rotor stiffness

One might choose the overall rotor parameters like the internal damping in the rotating frame, as $\mu = \frac{R_i}{2}$ such that its effective value in all directions is 'R_i'. The external damping of the system is taken as $\alpha = \frac{R_a}{2}$ such that the effective damping coefficient in all direction is 'R_a'. The stiffness coefficient $\varepsilon = \frac{K_s}{2}$, such that '$K_s$' is experienced in all directions. If the overall dynamics of the rotor is considered then the dynamic equation can be given as below

$$m\begin{bmatrix}\ddot{X}\\ \ddot{Y}\end{bmatrix} = -K\begin{bmatrix}X\\ Y\end{bmatrix} - \begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}\dot{R}_1 + \begin{bmatrix}0 & \omega R_1\\ -\omega R_1 & 0\end{bmatrix}\begin{bmatrix}X\\ Y\end{bmatrix}$$

The above equation can also be written by the overall force components experienced by the rotor in 'X' and 'Y' directions of the shaft the governing equation in fixed reference frame can be seen from the equation below,

$$\begin{bmatrix}F_{xr}\\ F_{yr}\end{bmatrix} = -K\begin{bmatrix}X_f\\ Y_f\end{bmatrix} + R_1\begin{bmatrix}\dot{\dot{X}}_f\\ \dot{Y}_f\end{bmatrix} + \begin{bmatrix}0 & -\omega R_1\\ \omega R_1 & 0\end{bmatrix}\begin{bmatrix}X_f\\ Y_f\end{bmatrix}$$

The dynamic force components can be seen as simple stiffness dependent forces, ordinary damping force proportional to $\dot{X}_f$ and $\dot{Y}_f$ and lastly the last component of the force which is of special significance and its inherent nature may be discussed by the later parts of the section, This component of the force can also be called as the regenerative force or the circulating force which exhibits a special feature that it is solely responsible for creating instability in the rotor system due to its overall anti symmetry by nature.

One might take this peculiar asymmetric force component given below for further analysis.

$$\begin{bmatrix} F_{xc} \\ F_{yc} \end{bmatrix} = \begin{bmatrix} 0 & -\omega R_1 \\ \omega R_1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}$$

the special characteristics of this circulating force component is that it cannot be actually derived from any potential components. One may then say that like most of the other force components found in nature this force is apparently non-potential $\bar{F}_c \neq -\bar{V}\Phi$ where the function $\phi(x, y)$ can be defined as some potential function.

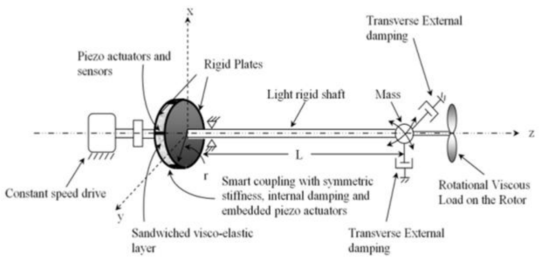


Fig. 2 Schematic representation of cylindrical journal bearing with fixed (X,Y) coordinate system

III CONTROLLER DESIGN

Consider a non-linear control affine system given by

$\dot{x} = f(x) + g(x)u$

The task is to find a control input that minimizes the performance index given by

$J(x(t_0), t_0) = \Phi(x(T), T) + \int_{t_0}^{T} L(x(\tau), u(\tau))d\tau$

Along with boundary conditions

$x(t_0) = x_0$ fixed and $x(t_f)$ free.

The utility function L is given by

$$\Psi(x,u) \overset{\Delta}{\Rightarrow} \frac{1}{2}(x^T Q x + u^T R u)$$

let us define a scalar function J*(x*(t),t) as the optimal value of the performance index J for an initial state x*(t) at time t, i.e.,

$J^*(x^*(t),t) = (x(T),T) + \int_t^T L(x^*(\tau), u^*(\tau), \tau)d\tau$

Consider a Hamiltonian given by

$H(x, \lambda^*, u) = L(x,u) + \lambda^{*T}[f(x) + g(x)u]$

where $\lambda^* = \frac{\partial J^*}{\partial x}$ .The optimal control is obtained from the necessary condition given by

$\frac{\partial H}{\partial u} = \frac{\partial L}{\partial u} + \lambda^{*T} \frac{\partial}{\partial u}[f(x) + g(x)u] = 0$(3.7)

This gives the following optimal control equation for control affine system described in (3.2):

$u = -R^{-1} g^T \lambda^*$

Substituting the value of u into (6.5), we get

$H(x^*, \lambda^*, u^*) = \frac{1}{2} x^{*T} Q x^* + \frac{1}{2} \lambda^{*T} g x^* R^{-1} g^T \lambda + \lambda^{*T}[f - g R^{-1} g^T \lambda^*]$

On simplification, we have the following optimal Hamiltonian:

$H^* = \frac{1}{2} x^{*T} Q x^* - \frac{1}{2} \lambda^{*T} g R^{-1} g^T \lambda^* + \lambda^{*T} f$

$= \frac{1}{2} x^{*T} Q x^* - \frac{1}{2} \lambda^{*T} G \lambda + \lambda^{*T} f$

Where $G = gR^{-1}g^T$.We know that the optimal vale function J*(x*,t) must satisfy the HJB equation given by

$$\frac{\partial J^*}{\partial t} + min_u H\left(x, \frac{\partial J^*}{\partial x}, u, t\right) = 0$$

With the boundary condition given by

$$J^*(x^*(T), T) = \Phi(x^*(T), T)$$

It provides the solution to the optimal control problem for general nonlinear dynamical systems. However, the analytical solution to the HJB equation is difficult to obtain in most cases. It is well known that the HJB equation is both necessary and sufficient conditions of optimality. Therefore, by combining (3.2) and (3.11), we can say that, in case of control affine systems(3.11), the optimal value function must satisfy the following nonlinear dynamic equation:

$\frac{\partial J^*}{\partial t} + \frac{1}{2} x^{*T} Q x^* - \frac{1}{2}\left(\frac{\partial J^*}{\partial x}\right)^T G \frac{\partial J^*}{\partial x} + \left(\frac{\partial J^*}{\partial x}\right)^T f = 0$

Since, the analytical solution of the above equation is difficult, we take a different approach and approximate the optimal value function as follows:

V(x, t)=h(w, x)

Where the approximating function h(w, x) is selected so as to satisfy certain initial conditions stated in the next section. The parameter t has been put in V(x, t) to show explicit dependence of the value function on time because of time varying parameters w in the approximating function h(w, x).

For the value of function given in (3.12) to be optimal, it must satisfy the HJB

$$\frac{\partial V}{\partial t} + L(x,u) + \left(\frac{\partial V}{\partial x}\right)^T [f + gu] = 0$$

$\frac{\partial h}{\partial w}\dot{w} + \frac{1}{2} x^T Q x - \frac{1}{2}\left(\frac{\partial V}{\partial x}\right)^T G \frac{\partial V}{\partial x} + \left(\frac{\partial V}{\partial x}\right)^T f = 0$

This gives the following weight update law:

$$\frac{\partial h}{\partial w}\dot{w} = -\frac{1}{2}x^TQx + \frac{1}{2}\left(\frac{\partial h}{\partial x}\right)^T G\frac{\partial h}{\partial x} - \left(\frac{\partial h}{\partial x}\right)^T f$$

The task is to find $\dot{w}$so that the above scalar equation is satisfied.

This is an under-determined system of linear equations with the number of equations less than the number of variables to be estimated. Though, there are infinitely many solutions for which $\dot{w}$ would exactly satisfy the above equation, we seek the one which minimizes $||\dot{w}||_2$. The problem is referred to as finding a minimum norm solution to an under-determined system of linear equations. The pseudo- inverse method is used to solve this problem.Eqn(3.15) may be written as

$$s\dot{w} = r$$

where s=$\frac{\partial h}{\partial w}$ is a 1*N_w vector and r= -$\frac{1}{2}x^TQx$+$\frac{1}{2}\frac{\partial h}{\partial x}^T G\frac{\partial h}{\partial x}$ - $\left(\frac{\partial h}{\partial x}\right)^T f$
is a scalar quantity.

The pseudo- inverse solution is given by
$\dot{w} = s^T(ss^T)^{-1}r$

Note that the term ss^T is a scalar quantity and its inverse is easily computable.

The control scheme is shown in figure 3.1. The blocks are self-explanatory.

Consider a linear system of the form by
$$m\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = -k\begin{bmatrix}x\\ y\end{bmatrix} - R_i\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} + \begin{bmatrix}0 & wR_i\\ -wR_i & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

$$\begin{bmatrix}\ddot{x}\\ \ddot{y}\end{bmatrix} = -\frac{R_i}{m}\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} + \frac{1}{m}\begin{bmatrix}0 & wR_i\\ -wR_i & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

From the above equations we get,
$$\ddot{x} = -\frac{R_i}{m}\dot{x} + 0 + \frac{wR_i}{m}y$$
$$\ddot{y} = -\frac{R_i}{m}\dot{y} + \left(-\frac{wR_i}{m}\right)x + 0$$

On solving eqn1 and eqn 2, we get,
$$\dot{x_1} = x_2$$
$$\dot{x_2} = -\frac{R_i}{m}x_2 + \frac{\omega R_i}{m}y_1 + u_1$$

$$\dot{y_1} = y_2$$
$$\dot{y_2} = -\frac{R_i}{m}y_2 - \frac{\omega R_i}{m}x_1 + u_2$$

State space model for the above equations:
$$\begin{bmatrix}\dot{x1}\\ \dot{x2}\\ \dot{x3}\\ \dot{x4}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & \frac{-R_i}{m} & \frac{wR_i}{m} & 0\\ 0 & 0 & 0 & 1\\ -\frac{wR_i}{m} & 0 & 0 & \frac{-R_i}{m}\end{bmatrix}\begin{bmatrix}x1\\ x2\\ y1\\ y2\end{bmatrix} + \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ u1\\ 0\\ u2\end{bmatrix}$$

The initial values are
m= 0.1kg ,
$R_i = 0.2$,
internal damping=0.02
w=660 rad/sec

$$\begin{bmatrix}\dot{x1}\\ \dot{x2}\\ \dot{x3}\\ \dot{x4}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -2 & 1320 & 0\\ 0 & 0 & 0 & 1\\ -1320 & 0 & 0 & -2\end{bmatrix}\begin{bmatrix}x1\\ x2\\ y1\\ y2\end{bmatrix} + \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\ u1\\ 0\\ u2\end{bmatrix}$$

Here,
$$A=\begin{bmatrix}0 \quad 1 \quad 0 \quad 0\\ 0-2 \quad 1320 \quad 0\\ 0 \quad 0 \quad 0 \quad 1\\ -1320 \quad 0 \quad 0 \quad -2\end{bmatrix}$$
$$\&B=\begin{bmatrix}0\,0\,0\,0\\ 0\,1\,0\,0\\ 0\,0\,0\,0\\ 0\,0\,0\,1\end{bmatrix}$$

The task to find the control law **u=c(x)**that minimizes the cost function
$$J = \frac{1}{2}\int_0^\infty [x^TQx + u^TRu]dt$$
$$Q = \begin{bmatrix}1\,0\,0\,0\\ 0\,1\,0\,0\\ 0\,0\,1\,0\\ 0\,0\,0\,1\end{bmatrix} and R = 1$$

The optimal value function for the given linear system is given by,
$$V = \frac{1}{2}(w_1x_1^2 + w_2x_2^2 + 2w_3x_1x_2 + w_4y_1^2 + w_5y_2^2 + 2w_6y_1y_2)$$

On differentiating V with respect to $x_1, x_2, y_1 and y_2$ we get
$$\frac{\partial V}{\partial x} = \begin{bmatrix}w_1x_1 + w_3x_2\\ w_2x_2 + w_3x_1\\ w_4y_1 + w_6y_2\\ w_5y_2 + w_6y_1\end{bmatrix}$$

Taking transpose of $\frac{\partial V}{\partial x}$;
$$\frac{\partial V^T}{\partial x} = [w_1x_1 + w_3x_2w_2x_2 + w_3x_1w_4y_1 + w_6y_2w_5y_2 + w_6y_1]$$

Again on differentiating V with respect to $w_1, w_2, w_3, w_4, w_5 and w_6$ we get;
$$\frac{\partial V^T}{\partial w} = [0.5x_1^2 \quad 0.5x_2^2x_1x_2 \quad 0.5y_1^2 \quad 0.5y_2^2y_1y_2]$$

The derivative of the weight vector $\dot{\boldsymbol{w}}$is obtained by solving weight update law:
$$\frac{\partial V}{\partial w}\dot{w} = -\frac{1}{2}x^TQx + \frac{1}{2}\frac{\partial V^T}{\partial x}B\frac{\partial V}{\partial x} - \frac{\partial V}{\partial x}Ax$$

A control law is a mathematical formula used by the controller to determine the output '**u**' that is sent to the plant. In a feedback control scenario, the output u can lead to robustness to uncertainty, and can be used to design system dynamics. A control law, which is usually part of the controller, takes the output of a process one wants to control as input. This input is then used in an algorithm that the controller uses to determine u, which is the input to the system (plant). For example, in a cruise control system, the control law compares the reference speed to the current speed and uses this error (vr - v) in a PID control algorithm to determine the signal u that is then sent to the actuators to decrease the error between the desiredand actual speed.

The control law for this problem, it is compounded to be

$$u = -R^{-1}B^{T}\frac{\partial v}{\partial x}$$

IV SIMULATION

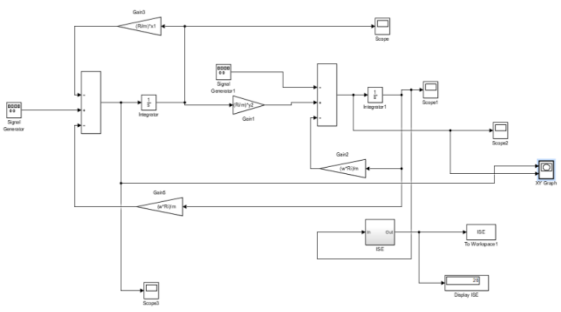


Fig 3 Simulink Model of micro size Rotor

A . Dynamics of Uncontrolled Rotor System:

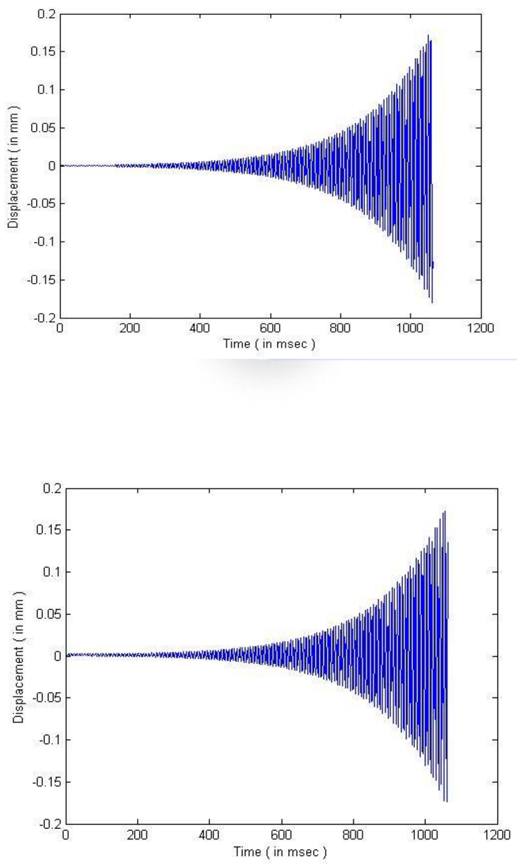
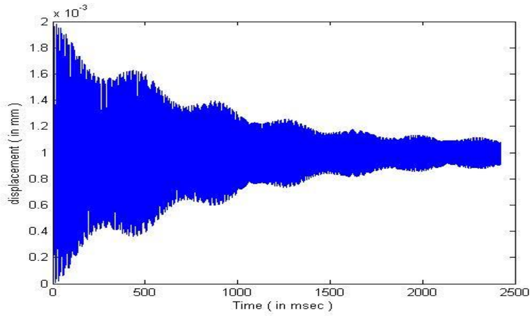
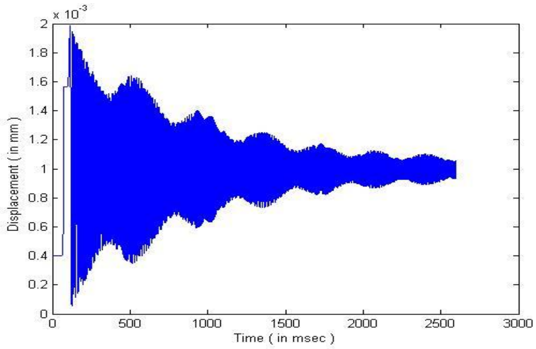


Fig 4 Instability of the micro rotor without the controller

B. Stabilization of micro rotor system Using Pontryagin (Optimal Controller)





V CONCLUSION

The paper introduces the stabilization of small size rotor using Pontraygrin optimal controller. This project proposes the stabilization strategy which is more suitable for mini-micro sized rotors, hydrodynamic bearing and short journal bearing. We have designed models when the system is fixed and unstable, after that we converted it to rotating frame which is also unstable. All the simulation results are also presented in support of this conclusion. We have used MATLAB for our working and analysis.

The proposed nature of the stabilizing control is algorithmic simplicity, there is effective Usage of memory, easy to deploy on a processor, mathematically, it is easy to comprehend, it has good speed response and it is a reduced order efficient controller design.

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