Probability distributions for design wave predictions in the Bay of Bengal

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***Abstract*—Design wave parameters are predicted using probability distributions and statistical methods.Future values are estimated from collected preceding sequence of values. In this papersignificant wave height (SWH) data are used to analyze and predict extreme wave events that can affect coastal based life and impact marine industries. Two locations are chosen at the Bay of Bengal (BOB) region for extreme height estimation for the period 1958-2001. In this study, Generalized Pareto distribution (GPD), Generalizedextreme value distribution (GEV) and Weibull distribution have been applied to approximate design wave parameters and examine extreme wave elevations for specified return periods. For the two sites at BOB and three probability distributions the expressions for the extreme value of wave height for given return periods, mean maximal wave height and most occurring wave heights have been formulated and then compared.**

Keywords—design wave parameters, return period, extreme wave heights, probability distributions, Bay of Bengal

I. INTRODUCTION

Time series data collected sequentially and originating from various natural processes usually contain useful information applicable to many fields. Probability distribution models are applied to signals obtained from time series to obtain information that is not readily available in raw form. Time-series analysis often considers the long-term trends and variations of individual (or groups of) parameters (or indicators). These are used to extrapolate future behavior. Usually, the pre-processing of the time-series parameters (predictors) is of secondary importance to building a predictive or explanatory model.

The Indian Ocean plays a foremost part in the worldwide financial growth and is the midpoint of global geopolitics. There is rapid progress in the various coastal and offshore activities. This includes construction of major ports and harbors, establishments of power plants and oil and gas industries. For the normal functioning and smooth operation of such developments along the coast various coastal and offshore structures are required to be constructed. Thus for the sustainability of these structures estimation of design wave parameters are required.

Wind seas are generated locally and are strongly coupled to the local wind field whereas swells with longer wavelengths are generated remotely and are not directly coupled to the local wind field. Since sea states are a combination of surface waves and distant storms, the wave energy spectra often contain two or more peaks. Multi-modal sea states have a significant impact on the design and operability of fixed and floating offshore structures. Thus arises the importance of the probability of occurrence of spectra with more than one peak for some given location. Swells in the ocean can often be surprisingly destructive which can cause serious damage to ships. After being generated by a storm, swell waves can propagate very long distances with little attenuation until they break and dissipate upon reaching a coast. How the swell propagates in the open ocean is an important factor while predicting ocean waves. It has application in research on global climate change, wave energy development, and disaster prevention and reduction. One can define wave height as the distance from the crest of one wave to the trough of the next wave. However, WMO defines significant wave height as "the average height of the highest one-third of the wave heights (sea and swell) occurring in a particular time period.

MetOcean Solutions in collaboration with the New Zealand Defence Force moored a buoy about 400 miles south of New Zealand. It recorded a monster wave on May 20, 2017, which was nearly 20 meters high. The measurement of the wall of water was 19.4 meters or approximately 63.6 feet whereas the significant wave height measured was 10.4 meters or approximately 34 feet during the same time period. Although this is one of the largest waves recorded in the Southern Hemisphere, the WMO considers "significant wave height" to be the official measure of the largest sea state. The highest significant wave height recorded remains 19 meters (62.3 feet) measured by a buoy in the North Atlantic.

Between Iceland and the United Kingdom, on 4 February 2013 in the North Atlantic Ocean, the above wave was recorded by an automated buoy at 0600 UTC having location approximately 59° N, 11° W. A very strong cold front passed after this which resulted in very high winds (50.4 miles per hour) over the area. The previous record of 18.275 meters (59.96 feet) was measured on 8 December 2007, also in the North Atlantic. Thus to protect the lives of crew and passengers on busy shipping lanes and ensure the safety of the global maritime industry ocean observations and forecasts leading to such measurements are very important. Compared to the Southern Ocean the typical highest waves are found to occur in the North Atlantic

The international scientific community with the aid of satellite imageries is aware that the monster waves are not rare events. The European Space Agency tasked two of its earth scanning satellites to monitor the oceans with their radars as part of the project 'Maxwave' set up to test the existence of giant waves. During a three week period they detected ten abnormal waves of over 25m high. Over the last two decades, more than two hundred super carriers-cargo ships over 200m long have been lost at sea. Reports of people present suggest that many were sunk by high and violent walls of water that rose up out of calm seas. Two large ships sink every week on average. Unfortunately by existing techniques, a return period of ten thousand years is required for such an extreme wave height to occur and thus the development of freak waves is not clear.

It is to be noted that open ocean going vessels are designed to withstand a maximum height of 15m. It is well known that extreme waves are produced when wave propagation is opposed by a strong current. Many large ships have encountered problems in the regions where the Agulhas current going south meets with the swells from the Antarctic Ocean. The waves from storms in the open oceans are subjected to refraction and diffraction in shallower waters and hence there may be focusing of wave energy in certain areas. The probability of encountering large waves in these regions will be greater compared to other locality. But it is difficult to explain the extreme waves occurring in the open oceans. The possible physical causes may be due to time-space focusing, current focusing and nonlinear focusing.

Freak waves cannot be predicted using spectral wave models. Muraleedharan et al (2007b)studied the possibility of using extreme value distributions for extreme wave analysis. In the above study the modified Weibull model effectively reproduced the daily maximum significant wave height distributions during southwest monsoon season and for a cyclonic condition which is the initial requirement to be satisfied before any extreme wave analysis. They discussed the truncated Gumbel model was not able to simulate the wave height distribution for a cyclonic sea condition and the three parameter generalized Pareto distribution completely failed in explaining the different sea states. As per the study the extreme wave heights predicted for different return periods by parametric relations derived from modified Weibull distribution proclaimed that the deep water oceanic region off Goa has little chance to develop a freak wave. Under the global warming and associated changes in the wind pattern scenario, the methodology prescribed in the above study may be applied for various oceanic regions to pinpoint abnormal wave generating areas.

High waves accompanied by strong winds become risk factors for the ocean and naval activities and maritime industries. Zhang and Li (2017) studied dangerous sea states triggering ship accidents. They analyzed a 10 year (2001–2010) ship accident dataset from the International Maritime Organization including 3648 ship accidents. Numerical wave model generated parameters like significant wave height, mean wave period, and mean wave direction, were analyzed for the selected ship accident cases. With 1561 cases with exact geographical locations remaining in the dataset and the study focusing on the cases that occurred in swell-related sea states only 58 cases were retained. The analysis of the 58 swell-related accidents indicated that 52% of the cases occurred in relatively low sea state conditions with significant wave height values smaller than 3m. In these situations, the swell waves provided the major wave energy. Further analysis of these accidents suggested that when wind-sea and swells occur simultaneously, especially when the differences in their mean wave periods and mean wave directions meet certain conditions, there may be hazardous seas that generate risk to shipping activities. They concluded due to combined sea and swell conditions sailing vessels are subjected to risks, especially when there is similar wave periods and also oblique wave directions.

A crossing sea is a sea state having two wave systems traveling at oblique angles. In the past, a few studies have indicated that the occurrence of extreme waves and serious ship accidents (Bruns et al., 2011; Cavaleri et al., 2012) is correlated with the crossing sea state. Studies related to crossing seas highlighted the interaction between wind waves and swell waves, but space observations confirm after the crossing of two swell trains there is a marked change in the swell dissipation (Li et al., 2008).

The study of significant wave parameters having random characteristics are used for analyzing ocean activities and marine developments (Goda, 1997). As described by Longuet-Higgins (1952) nil up-crossing wave height distribution is given by Rayleigh density function having a scale parameter. Prevosto et al (2000) offered numerous small and lasting probability models for ocean wave height distribution. Draper (1970) and Ploeg (1968) highlighted the prominence of a decent approximation of the wave parameters for the construction of engineering and maritime structures. For the strategy of coastal structures long term information of extreme wave conditions is very much needed (Draper, 1973). Panchang et al (1999) discussed the difficulties in using the satellite based wave height data and Goldsmith et al (1983) the significance of optical ship comments and maneuvering upsurge device amounts. Laing (1985) established how visually reported wave statistics could be useful in meteorological works. For the estimation of the design wave parameters Soares (1986) described that graphic interpretations of upsurge heights were the chief basis of material obtainable for the forecast of extreme wave circumstances.

Caires (2011) thoroughly associated the frequently used extremum value distributions similar to GEV and GPD with altered parameter estimation approaches.The Weibull distribution prototype for wave elevation pattern is tinted in expressions of the shape and strength function (Muraleedharan et al 1991, 1993).Correction coefficients assimilated in the altered Weibull distribution showed to be more real for maximum wave height reproduction (Muraleedharan et al 2007a).

In the present study, different probability distribution functions have been used to evaluate design wave parameters for the BOB area. The difference in the extreme value estimates from ERA-40 analyzed ocean wave dataset for different distributions are considered and examined.

II. MATERIALS AND METHODS

The following two locations are chosen for study at the BOB region,(i) Central Bay having longitude 90E, latitude 15N and (ii) Head Bay having longitude 90E, latitude 20N. SWH data is downloaded from theEuropean Centre for Medium Range Weather Forecasts from1958-2001 (44 years). The spatial resolution of the dataset is one degree by one degree. The temporal resolution is 06 hours. The period 1958-1967 is chosen as the training period.The estimation models used in this paper to get extremum wave return values include the GEV, GPD and Weibull Distribution. "Fig. 1",shows the cumulative probability distribution of the Central grid SWH data and Head grid SWH data for the training period 1958-1967.

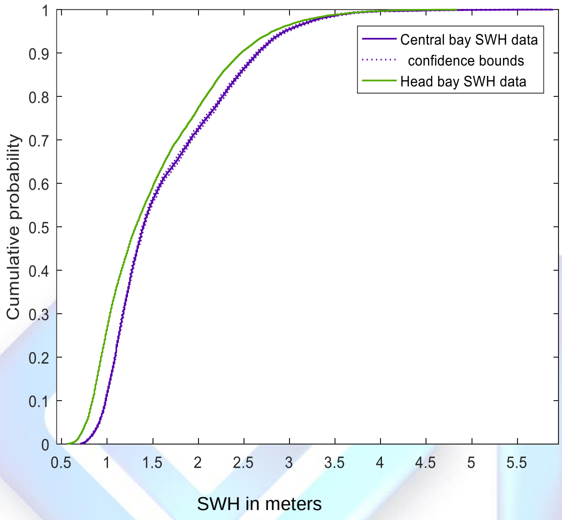

Fig. 1. Cumulative Probability Distribution of the Central grid SWH dataand Head grid SWH data for the period 1958-67.

Recorded (Histogram) and theoretical SWH data for the Central and Head Bay for GEV is depicted in "Fig. 2".

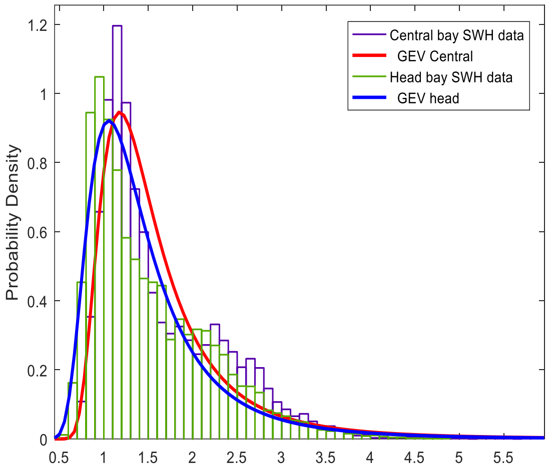

Fig. 2. Recorded (Histogram) and theoretical SWH data for the Central and Head Bay for GEV

Recorded (Histogram) and theoretical SWH data for the Central and Head Bay for GPD is depicted in "Fig. 3".

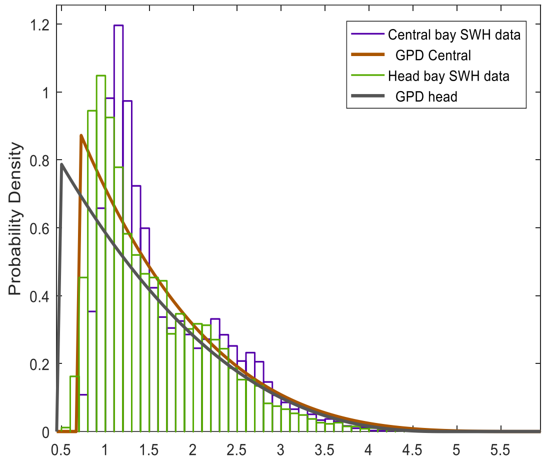

Fig. 3. Recorded (Histogram) and theoretical SWH data for the Central and Head Bay for GPD

Recorded (Histogram) and theoretical SWH data for the Central and Head Bay for Weibull Distribution is depicted in "Fig. 4".

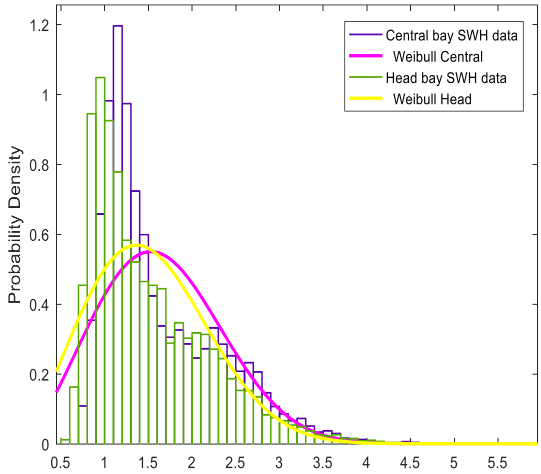

Fig.4. Recorded (Histogram) and theoretical SWH data for the Central and Head Bay for Weibull Distribution

III. RESULTS AND DISCUSSIONS

The parameters of GEV, GPDandWeibull Distribution are computed for the long-term sequence of wave heights. For, GEV Distribution the scale parameter σ, the location parameter μ,and the shape parameter ξ are 0.404911, 1.27821 and 0.296959 respectively for Central bay data. The samefor head bay are calculated as 0.4131, 1.15131 and 0.272192respectively.“Fig. 5” and “Fig. 6” gives the probability of occurrences of extreme wave heights for GEV and GPD respectively.

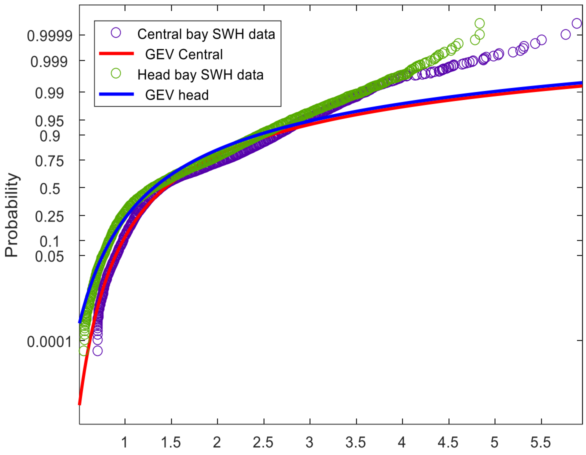

Fig. 5. Probability occurrences of extreme wave heights for GEV

For GPD, the parameters obtained from the data from the central bay are calculated as 1.12799, 0.7 and -0.21533 respectively.The parameters obtained from the data from the head bay are calculated as 1.40253, 0.4 and -0.3145 respectively.

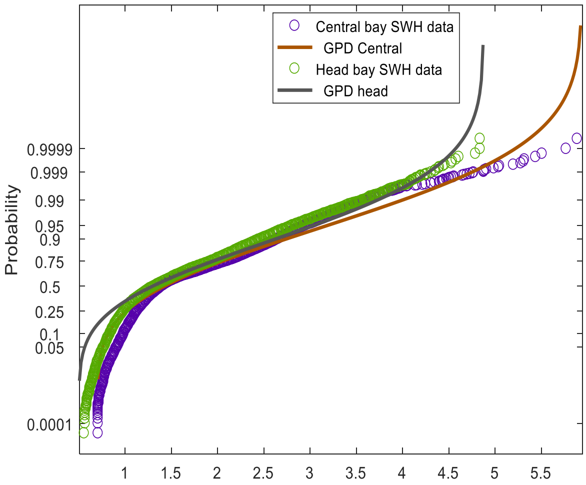

Fig. 6. Probability of occurrences of extreme wave heights for GPD

For Weibull Distribution the a-scale, b-shape parameters are 1.86095 and 2.54238 for central bay data set respectively. For head bay dataseta-scale and b-shape parameters are 1.71362 and 2.38846 respectively.

The maximal wave height calculated for prescribed return periods are estimated in the following tables:

TABLE I. EXTREME WAVE HEIGHTS FOR CENTRAL BAY SWH DATA

		GEV	GPD	Weibull
Return period	Computed	Expected	Expected	Expected
5	4.03	2.591	5.457	4.79
10		2.601	5.52	4.91
15		2.605	5.56	4.98
20		2.609	5.58	5.03
25		2.611	5.6	5.07
30		2.612	5.61	5.09
	RMSE	1.58	1.45	0.9

TABLE II. EXTREME WAVE HEIGHTS FOR HEAD BAY SWH DATA

		GEV	GPD	Weibull
Return period	Computed	Expected	Expected	Expected
5	3.85	2.743	4.72	4.69
10		2.729	4.75	4.82
15		2.72	4.76	4.89
20		2.718	4.77	4.94
25		2.715	4.77	4.98
30		2.713	4.78	5.02
	RMSE	1.3787	0.8054	0.9021

Extreme wave heights in the central bay (Table 1) estimated by Weibull distribution seems to be more close to the computed values which are followed by GPD and GEVdistributions. While forhead bay data the extreme wave heights (Table 2) estimated by GPD seems to be closer to the computed values followed by Weibull distribution and GEVdistribution.

Graphs pertaining to the expected values of the extreme values have been shown in “Fig. 7” and “Fig. 8”.

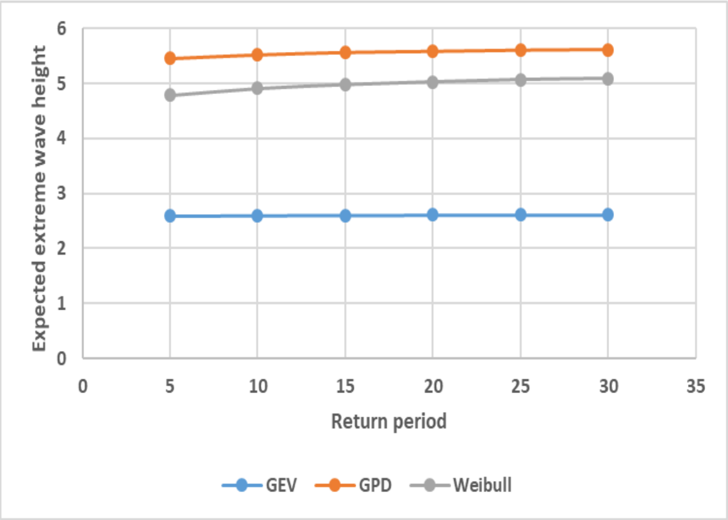

Fig. 7. Graph depicting the effect of return period on the extreme values for the different models for the central bay SWH data

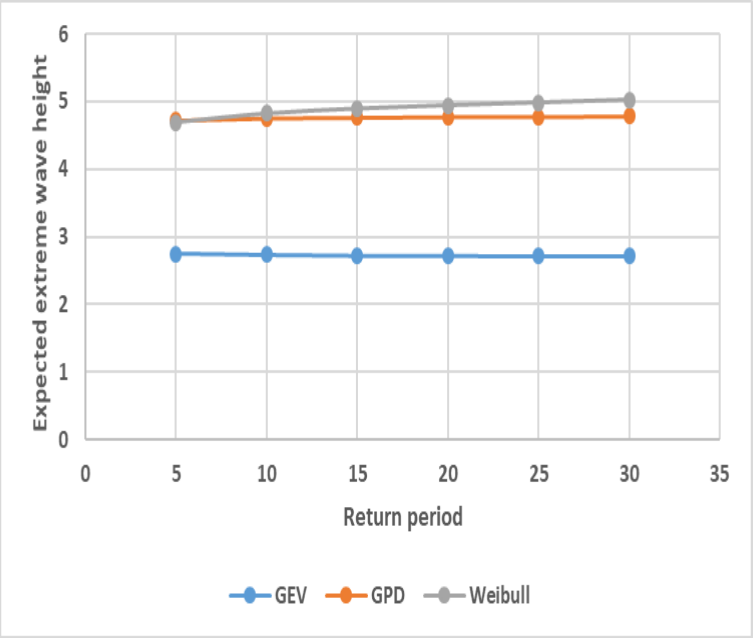

Fig.8. Graph depicting the effect of return period on the extreme values for the different models for the head bay SWH data

Next the mean maximalwave height and the most occurringmaximalwave height are estimated in Table 3.

TABLE III MEAN MAXIMAL WAVE HEIGHT AND THE MOST OCCURRING MAXIMAL WAVE HEIGHT

	Design Wave Heights (m)		GEV Expected	GPD Expected	Weibull Expected
		Computed			
Central Bay Data	Hmax	3.06	2.35	3.56	2.04
	Hmfm	1.1	1.44	1.42	0.65
Head Bay Data	Hmax	3.03	2.03	2.09	1.9
	Hmfm	0.86	1.32	1.26	0.61

“Fig. 9” and “Fig. 10” depict the change in the values of the mean maximal wave height and most occurring maximal wave height for the central and head bay SWH data respectively. For both central and head bay data the mean maximal wave height (Table 3) estimated by GPD is closer to the computed values than the GEV and Weibulldistribution. On the other hand, most occurring maximal wave height for central bay data(Table 3) estimated by GPD is closer to the computed value which is followed by GEV and Weibull Distribution. Whereas for head bay data, most frequent maximum wave height estimated by Weibull Distribution is closer to the computed value than GPD and GEV.

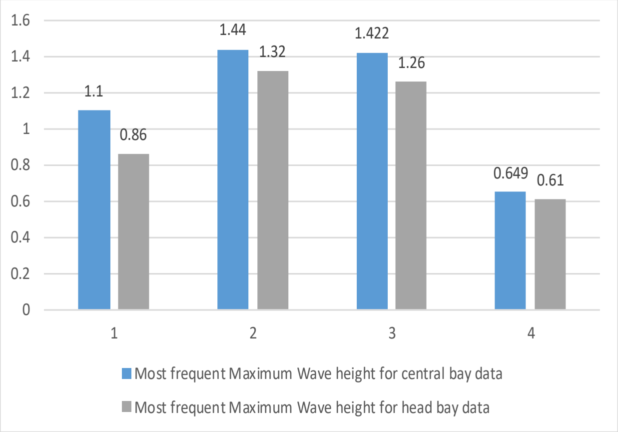

Fig. 9. Graph depicting the change in the values of the most frequent maximum wave height for the head bay SWH data

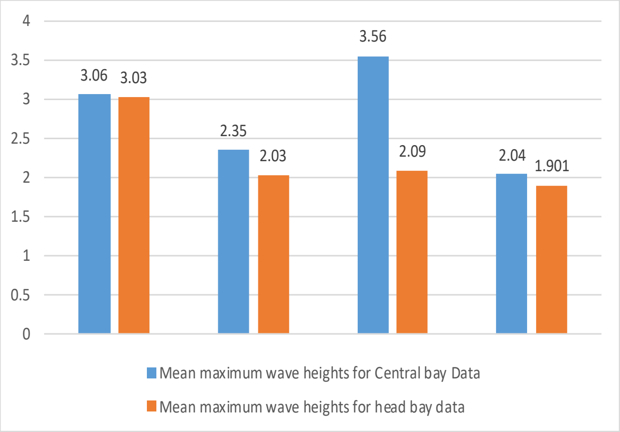

Fig. 10. Graph depicting the change in the values of the mean maximum wave height for the central bay SWH data

Thus from the outcomes, it is perceived that the extreme value approximations from the central bay data seem to deviate from GEV and GPD as compared to Weibull Distribution. However, for the headbay,GPD estimation seems to be more accurate than the other distributions. The estimation done using the GEV distribution has caused an underprediction, which suggests that high wave charts are sometimes difficult to capture. It is a acquainted occurrence and task that the flattening consequence entrenched in the mathematical replicas will lead to the compressed erraticism resulting in missing peaks. In addition, it is likely that since sampling rates are lower, the extreme wave heights in a tempest that occurs amid observations will not be documented.

IV. CONCLUSIONS

Time series examination is the assembly of data at specific intervals over a period, with the purpose of identifying trends, cycles, and seasonal variances to aid in the forecasting of a future event. Ability to predict future events diminishes, as measurements are done at random intervals. The most important objective of time series analysis is extrapolating past behavior into the future.

This paper concentrated on the assessment of the design wave parameters. The evaluationconducted and outcome attained will help in the construction of a long termmaximal wave chart for the study region, which may aid as a fastescort to recognizemost vulnerable seaside areas.Three different probability distributions have been used toestimate the return value assessment: the GEV model, the GPD model,andWeibullDistribution. It can be concluded that the Weibull distribution have accurately estimated the extreme wave height for various return periods for the central bay and GPD for the head bay.Nevertheless, all of them have their own benefits and inadequacies.

The main drawback of the GPD and GEV approaches are the extraordinary difference in undervaluing or overvaluing return values with respect to the obtained maximal values in the time series.

Hence, future scope lies in modifying the distributions and incorporating calibration coefficients in the modified models in order to be additionally operational for estimating the extreme wave parameters.

ACKNOWLEDGMENT

The authors are thankful to European Centre for Medium-Range Weather Forecasts (ECMWF) for the data sets.

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