TTIC, 2020, Vol. 4, 69-73

An Improved MUSIC Algorithm for DOA Estimation of Near-Coherent Quasi-Static Sources

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Abstract—**In this paper, we propose an improved subspace based method, analogous to MUltiple SIgnal Classification algorithm for direction-of-arrival estimation of quasi-static sources in the far field of the receiver antenna array. The data matrix of the standard MUSIC algorithm is conjugate transformed prior to the eigenvalue decomposition process. Simulation results of the pseudospectrum shows that better resolution is achieved in direction finding of near correlated sources for the improved method. Simulations are also performed in terms of root-mean-square-error and the results are compared with the standard MUSIC and modified MUSIC algorithms.**

Keywords—DOA estimation, MUSIC algorithm, subspace based reconstruction, probability of resolution

I. Introduction

One of the most significant aspect of array signal processing is to determine the direction of plane waves impinging on an array of sensors or antennas. The plane waves or signals are assumed narrowband and either originate from sources located in the far field region or reflected signals from targets, originating from a radar. The narrowband signals travel through a homogenous, isotropic medium and impinge on an array of antennas or sensors from various directions in the broadside. Direction-of-arrival (DOA) estimation is a signal processing method to find or determine the direction of the impinging signals (and hence location of the sources or targets) in terms of elevation and/or azimuth angles. DOA estimation problem has attracted researchers of radar, sonar and wireless communication for decades in developing more accurate with less complex algorithms.

Most significantly, a linear array of antennas with uniform spacing between the elements are used as the sensor array, constituting the uniform linear array (ULA). This structure is indispensable for determining either the elevation or the azimuth angles (1D DOA estimation). The elements of the ULA are assumed omnidirectional and isotropic, to evade any electromagnetic effect or mutual coupling. A real sensor noise element that consists of a white Gaussian noise component that is statistically independent from sensor to sensor is included in the structure.

Over the years, the method of spectral estimation generally referred to as the ability to select various frequency components out of a collection of signals. This concept was expanded to include frequency-wavenumber problems and subsequently DOA estimation. One of the earliest applicable development of DOA estimation was the Pisarenko harmonic decomposition (PHD) method [1, 2]. The objective is to minimize the mean-squared error of the array output under the constraint that the norm of the weight vector be equal to unity. The eigenvector that minimizes the mean squared error corresponds to the smallest eigenvalue. The signal and noise subspace is associated with two eigenvectors.The conditions of PHD was extended splendidly [3]to MUltiple SIgnal Classification (MUSIC) as a high-resolution eigenstructure method. MUSIC is a subspace method, as it exploits the noise subspace. As in other classical approaches, a spatial covariance matrix is formed from the signal components received from a dense uniform array of sensors or antennas.The noise and signal subspaces are separated by the eigenvalue decomposition of the spatial covariance matrix, which is projected by averaging over multiple snapshots. MUSIC assumes that the noise in each channel is uncorrelated making the noise correlation matrix diagonal. The incident signals may be somewhat correlated creating a non-diagonal signal correlation matrix. However, under high signal correlation the traditional MUSIC algorithm breaks down. A variant of the MUSIC algorithm, namely the root-MUSIC method [4], makes use of eigenvectors of the noise subspace and forms a complex polynomial whose roots correspond to the DOAs of the signal-of-interest. A recent restructured version of MUSIC, namely the modified MUSIC (MMUSIC) algorithm [5, 6]uses spatial smoothing by Nyström Approximation. This method enhances the resolution capabilities of practically correlated targets.

Another signal subspace based method that assumes and uses narrowband signals impinging on the array (to recognise the translational phase relationship between the multiple arrays) is the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [7] .The aim of the ESPRIT technique is to exploit the rotational invariance in the signal subspace that is created by two arrays with a translational invariance structure.

All conventional DOA estimation methods suffer from serious drawbacks particularly, in terms of low resolution of correlated signals, estimation at lesser SNR values, fewer degrees of freedom (DOF), high computational complexity and multiple snapshot requirements, among others [8, 9].

In recent years, non-uniform arrays and sparse reconstruction based optimization has taken centre-stage in DOA estimation research. By single snapshot, these methods provide more DOF, better resolution with lower complexity.

Nevertheless, MUSIC algorithm remain as a standard and fundamental procedure, as a building block to develop more complex but accurate methods. Its simplicity, higher degree of resolution for uncorrelated sources and somewhat less complexity makes it a suitable candidature for comparison with the recently developed algorithms.

In this paper, we modify the correlation matrix produced at the array output by each element through conjugate transformation, before performing the eigenvalue decomposition. The noise and signal subspaces are separated by the standard practise and the MUSIC like pseudospectrum is formed. Simulation results evidently shows the superiority of the improved method when compared with the standard MUSIC algorithm for estimation of near correlated signals. Performance of the new method is evaluated in terms of variation of root-mean-square-error (RMSE) and probability of resolution (P_{res}) with signal-to-noise ratio, (SNR)and the results are compared with standard MUSIC and modified MUSIC (MMUSIC) [5, 6] algorithms.

The rest of the paper is organized as follows. Section-II gives a brief description of the data model of DOA estimation and MUSIC algorithm. In section-III, development of the improved MUSIC algorithm is accomplished. The modified MUSIC algorithm is also discussed briefly in this section. Various simulation results and discussions constitute section-IV. The paper ends with a conclusion in section-V.

II. DATAMODEL OF DOA ESTIMATION

Fig 1 illustrates the schematic of a single narrowband plane

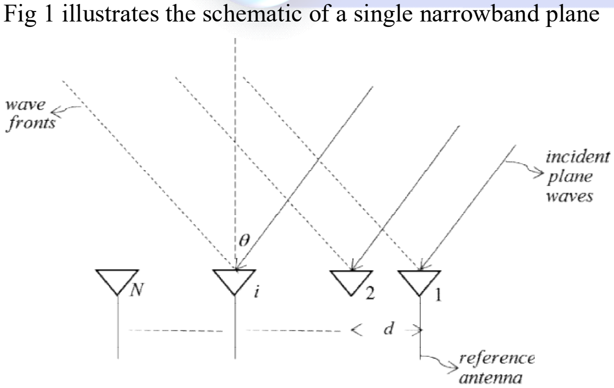

Fig. 1. Configuration of a uniform linear array for DOA estimation

wave originating in the far-field region, impacting on a uniform linear array (ULA) of N sensors. The sensors or the antenna elements are organised in a linear geometrical arrangement with uniform inter-element spacing of $d = \lambda/2$ (where λ is the wavelength of the signal with respect to its central frequency).This positioning is kept to achieve zero or minimum mutual coupling effect. θ is the direction of arrival (DOA) or angle of arrival (AOA) of the impinging signal in terms of azimuth or elevation angle. The aim of the direction-finding problem is to parametrically estimate the angle of arrival, θ.

The array factor (AF) of the ULA can be derived as [10],

$$AF = 1 + e^{j(kdcos\theta+\beta)} + e^{j2(kdcos\theta+\beta)} + \cdots e^{j(N-1)(kdcos\theta+\beta)} \quad (1)$$

where β is the phase difference between the elements and $k = \frac{2\pi}{\lambda}$ is the wave number. The AF is dependent on the geometric arrangement of the array elements, the spacing of the elements, and the electrical phase of each element.

The series of Eqn. (1) can be expressed as,

$$AF = \sum_{n=1}^{N} e^{j(n-1)(kdCos\theta+\beta)} = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad (2)$$

where$\psi = kdCos\theta + \beta$

In vector notations, Eqn. (1) and (2) can be represented as,

$$\overline{\boldsymbol{a}(\theta)} = \begin{bmatrix} 1 \\ e^{j(kd\cos\theta+\beta)} \\ \vdots \\ e^{j(N-1)(kd\cos\theta+\beta)} \end{bmatrix} \quad (3)$$

or,

$$\overline{\boldsymbol{a}(\theta)} = [1\ e^{j(kd\cos\theta)} \ \dots\dots\dots\ e^{j(N-1)(kd\cos\theta+\beta)}]^T \quad (4)$$

where$[\cdot]^T$signifies the transpose of the vector.

The vector $\overline{\boldsymbol{a}(\theta)}$is of the form of a Vandermonde vector and is known as *array steering vector*or simply *steering vector*. The AF can be expressed as the sum of the elements of the steering vector,$AF = sum(a(\theta))$.

Considering multiple plane waves originating at the far field region and impinging on the ULA from various directions (D) with DOAs $\theta_1, \theta_2 \dots\dots\dots \theta_D$, the received signal vector, comprising of the received signal of each antenna or sensor elements $x_i(k)$ is given as,

$$\overline{\boldsymbol{x}}(k) = \overline{\boldsymbol{A}}(\theta).\overline{\boldsymbol{s}}(k) + \overline{\boldsymbol{n}}(k) \quad (5)$$

where$\overline{\boldsymbol{x}}(k)$ is the vector of the received signals of each array element, of dimension $N \times 1$, given as $\overline{\boldsymbol{x}}(k) = [x_1(k)x_2(k) \ \dots\dots x_N(k)]^T$. Each received signal $x_i(k)$ also includes additive white Gaussian noise of zero mean value.$\overline{\boldsymbol{A}}(\theta)$ is the $N \times D$*matrix of the steering vectors* or the *array manifold matrix* or simply *steering* matrix of the ULA including the reference antenna given by,

$$\overline{\boldsymbol{A}}(\theta) = [\overline{\boldsymbol{a}}(\theta_1)\ \overline{\boldsymbol{a}}(\theta_2)\overline{\boldsymbol{a}}(\theta_3) \dots\dots\dots \overline{\boldsymbol{a}}(\theta_D)] \in \mathbb{C}^{N\times D} \quad (6)$$

$$\overline{\boldsymbol{s}}(k) = [s_1(k)s_2(k) \dots\dots s_D(k)]^T \quad (7)$$

is the vector of incident plane waves impinging on the ULA from D directions at time k.$\boldsymbol{n}(k)$ is the additive white Gaussian noise vector of dimension $N \times 1$, whose mean is zero and variance σ_n^2.

Thus, N elements of the ULA intercepts D complex signals impinging at angles θ_i, which have originated in the far field region. By tradition, $N > D$ is assumed. The modelling is based on time snapshots of the incoming signals as the signals received are time varying. If the targets or source locations or transmitters are moving, the matrix of the steering vectors will also vary, as the corresponding arrival angles changes.

Defining the $N \times N$ array correlation matrix of the received signal as $\bar{\boldsymbol{\mathcal{R}}}_{xx}$we have,

$$\bar{\boldsymbol{\mathcal{R}}}_{xx} = E[\bar{x}.\bar{x}^H] = E[(\bar{\boldsymbol{A}}.\bar{\boldsymbol{s}} + \bar{\boldsymbol{n}})(\bar{\boldsymbol{s}}^H.\bar{\boldsymbol{A}}^H + \bar{\boldsymbol{n}}^H)] = \bar{\boldsymbol{A}}E[\bar{\boldsymbol{s}}.\bar{\boldsymbol{s}}^H]\bar{\boldsymbol{A}}^H + E[\bar{\boldsymbol{n}}.\bar{\boldsymbol{n}}^H] \quad (8)$$

or,

$$\bar{\boldsymbol{\mathcal{R}}}_{xx} = \bar{\boldsymbol{A}}\bar{\boldsymbol{\mathcal{R}}}_{ss}\bar{\boldsymbol{A}}^H + \bar{\boldsymbol{\mathcal{R}}}_{nn} \quad (9)$$

where$\bar{\boldsymbol{\mathcal{R}}}_{ss} = D \times D$source correlation matrix

$\bar{\boldsymbol{\mathcal{R}}}_{nn} = \sigma_n^2\bar{\boldsymbol{I}}$is the $N \times N$ noise correlation matrix

$\bar{\boldsymbol{I}}$is the $N \times N$ identity matrix

If the sources and targets are uncorrelated then $\bar{\boldsymbol{\mathcal{R}}}_{xx}$ is a $N \times N$ Hermitian matrix, i.e. $\bar{\boldsymbol{\mathcal{R}}}_{xx} = \bar{\boldsymbol{\mathcal{R}}}_{xx}{}^H$ .Proficient eigenvalue decomposition of the correlation matrix divides the *space* into *noise* and *signal subspaces*. The larger eigenvalues correspond to the stronger signals and the smaller eigenvalues correspond to the weaker signals or noise. MUltiple SIgnal Classification (MUSIC) algorithm proposed in 1986 by Schmidt [3] is one of the most widely used method to estimate the DOA for uncorrelated signals and noises using the eigenvalue decomposition concept. The *pseudospectrum*, which is a graphical indication of the angles of arrival based upon maxima versus angle for MUSIC is derived as [3],

$$P_{MU} = \frac{1}{|\bar{\boldsymbol{a}}(\theta)^H\bar{\boldsymbol{E}}_N\bar{\boldsymbol{E}}_N^H\bar{\boldsymbol{a}}(\theta)|}$$

where $\bar{\boldsymbol{a}}(\theta)$ is the steering vector and $\bar{\boldsymbol{E}}_N$ is the noise subspace. The smallest eigenvalues correspond to noise variance provided the signals are uncorrelated. Hence, the $N \times (N - D)$ dimensional subspace spanned by the noise eigenvectors can be constructed such that,$\bar{\boldsymbol{E}}_N = [\bar{\boldsymbol{e}}_1\, \bar{\boldsymbol{e}}_2 \dots \dots \bar{\boldsymbol{e}}_{N-D}]$. (10)

III. IMPROVED MUSIC ALGORITHM

Under the proposition of a precise model, the MUSIC algorithm can theoretically achieve an arbitrarily high resolution to DOA estimation. However, the MUSIC algorithm is limited to direction finding of uncorrelated signals.When the sources are correlated or nearly correlated, the estimated performance of the MUSIC algorithm deteriorates or even completely loses.In addition, at low signal-to-noise ratio (SNR) levels, MUSIC fails to resolve even uncorrelated sources. In this section, we propose a modified mathematical approach of the standard MUSIC algorithm using conjugate transformation, which achieves better resolution in estimating near coherent or correlated sources.Also the modified MUSIC algorithm is briefly discussed here.

A transformation or transition matrix of M^{th} order is introduced as J anti-matrix, such that,

$$J = \begin{bmatrix} 0\ 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1\ 0 & \cdots & 0 \end{bmatrix}_{M \times M} \quad (11)$$

Let $\boldsymbol{Y} = \boldsymbol{J}\boldsymbol{X}^*$, where $\boldsymbol{X}^*$is the complex conjugate of $\boldsymbol{X}$. The covariance of the data matrix $\boldsymbol{Y}$can be given as,

$$\bar{\boldsymbol{\mathcal{R}}}_{yy} = E[\bar{\boldsymbol{Y}}.\bar{\boldsymbol{Y}}^H] \quad (12)$$

The reconstructed conjugate matrix is obtained from the summation of $\bar{\boldsymbol{\mathcal{R}}}_{xx}$and $\bar{\boldsymbol{\mathcal{R}}}_{yy}$, such that,

$$\bar{\boldsymbol{A}}\bar{\boldsymbol{\mathcal{R}}}_{ss}\bar{\boldsymbol{A}}^H + \boldsymbol{J}[\bar{\boldsymbol{A}}\bar{\boldsymbol{\mathcal{R}}}_{ss}\bar{\boldsymbol{A}}^H]^*\boldsymbol{J} + 2\bar{\boldsymbol{\mathcal{R}}}_{nn} \quad (13)$$

$$\bar{\boldsymbol{\mathcal{R}}} = \bar{\boldsymbol{\mathcal{R}}}_{xx} + \bar{\boldsymbol{\mathcal{R}}}_{yy}$$

Eqn. (13) replaces Eqn. (9) for eigenvalue decomposition and subsequent separation of noise and signal subspaces. The noise subspace is used to form the pseudospectrum of Eqn. (10) and obtain the estimated DOA value by finding the peak.

In MMUSIC algorithm, as proposed [5], a new array correlation matrix $\boldsymbol{R}_{xxN'}$ is formed from randomly chosen N' elements from N for $N' < N$. Next, Nyström approximation is employed for estimation of noise subspace, $E_{N'}$. After this, the procedure continues in a similar approach as MUSIC in estimating DOA angle θ.

IV. SIMULATION RESULTS

The simulations are performed in MATLAB R2018a on a laptop with Intel (R) Core (TM) i7-4790 CPU @ 3.60 GHz and 8 GB RAM. The system type is 64-bit operating system-Microsoft Windows 7 Home Premium.

A ULA of $N = 100$ isotropic, omnidirectional antenna elements is constructed along the $x-$ axis by using MATLAB. Two sources are assumed at the far field region and signals impinge on the ULA at $\theta_1 = 20^0$ and $\theta_2 = 60^0$ for uncorrelated case and $\theta_1 = 51^0$, $\theta_2 = 55^0$ respectively for nearly correlated case. Narrowband signals of centre frequency $3GHz$, travelling through homogenous medium impinge on the ULA. The inter-element gap between the elements is fixed at $d = \lambda/2$, where λ is the wavelength of the impinging plane waves. The total number of snapshots considered for all algorithms is 200.

Fig. 2 and Fig. 3 shows the estimation result of the standard MUSIC algorithm and improved MUSIC for the

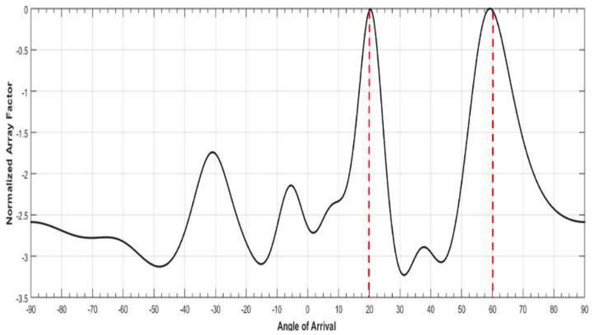

Fig. 2. DOA estimation by standard MUSIC algorithm for two uncorrelated sources located at $\theta_1 = 20^0$ and $\theta_2 = 60^0$. SNR=0 dB

uncorrelated sources at signal-to-noise ratio (SNR) value fixed at 0 dB. Fig. 4 and Fig. 5 depicts the direction finding capabilities of nearly correlated sources by the algorithms at SNR = 0 dB.

Simulation results on the estimation of location of nearly correlated sources are shown in Fig. 4 and Fig. 5, for standard and improved MUSIC respectively at SNR = 0 dB.

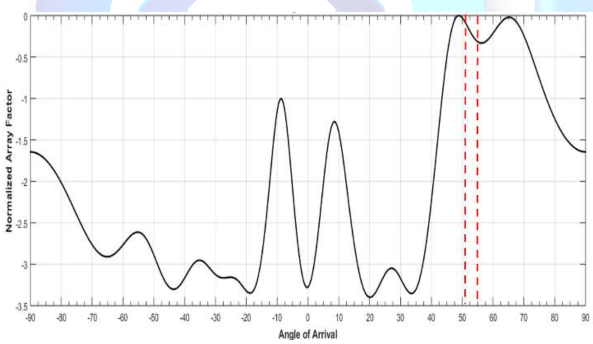

Fig. 4. DOA estimation by standard MUSIC algorithm for two nearly correlated sources located at $\theta_1 = 51^0$and $\theta_2 = 55^0$, SNR=0 dB

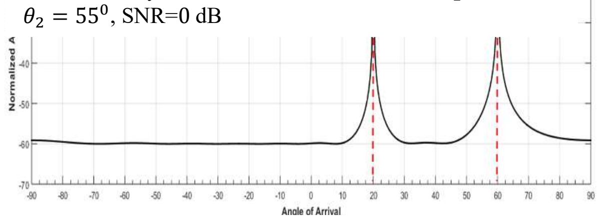

Fig. 3. DOA estimation by improved MUSIC algorithm for two uncorrelated sources located at $\theta_1 = 20^0$ and $\theta_2 = 60^0$, SNR=0 dB

From the plot of simulation results, it is clear that although MUSIC algorithm precisely identifies uncorrelated source directions, it fails convincingly as the sources become near-coherent. Whereas, in the case of improved MUSIC algorithm, both the uncorrelated and correlated sources are appropriately resolved.

The peaks in the graphs represent the simulated angles of arrival of the signal. The vertical red dotted lines denote the actual directions of arrival or the locations of the sources in the far field region.

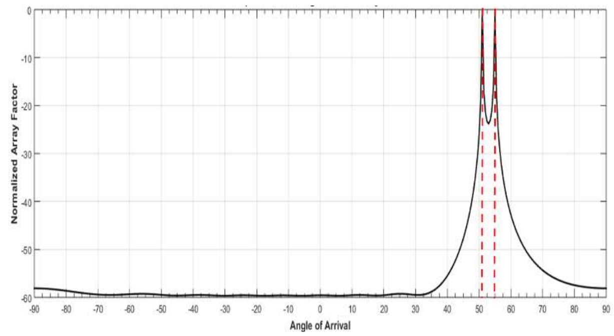

Fig. 5. DOA estimation by improved MUSIC algorithm for two nearly correlated sources located at $\theta_1 = 51^0$and $\theta_2 = 55^0$, SNR=0 dB

Next, the standard MUSIC, improved MUSIC and modified MUSIC algorithms are compared in terms of probability of resolution (P_{res}) and root-mean-square-error (RMSE) of the estimated DOA in terms with the SNR values. The probability of resolution (P_{res}) is defined as,

$$P_{res} = Prob\left\{\left|\hat{\theta}_i - \theta_i\right| \leq \frac{\Delta\theta}{2}\right\}, i = 1 \dots \dots m \quad (14)$$

[11]

where$\Delta\theta = \min\{|\theta_{i1} - \theta_{i2}|, 1 \leq i_1 \leq i_2 \leq m\}$.

The RMSE for Q number of Monte Carlo trials is given as [11],

$$RMSE = \sqrt{\frac{1}{Qm}\sum_{q=1}^{Q}\sum_{i=1}^{m}(\hat{\theta}_i - \theta_i)^2} \quad (15)$$

assuming$Q = 1000$ trials.

The SNR levels are varied between ± 15 dB.

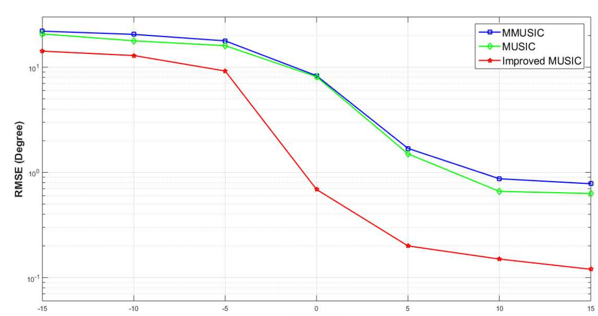

Fig. 6. Variation of RMSE of DOA estimation with SNR for standard MUSIC, MMUSIC and improved MUSIC algorithms

Fig. 6 depicts the simulated variation of root-mean-square-error with signal-to-noise ratio (SNR) levels for uncorrelated source locations. The proposed improved

MUSIC algorithm clearly outperforms the standard and modified MUSIC counterparts, both at low and high SNR values.

Fig. 7 shows the simulated variation of probability of resolution (P_{res}) with signal-to-noise ratio (SNR) values for uncorrelated source locations. The improved MUSIC algorithm performs better at lower SNR values, as observed from the plot.

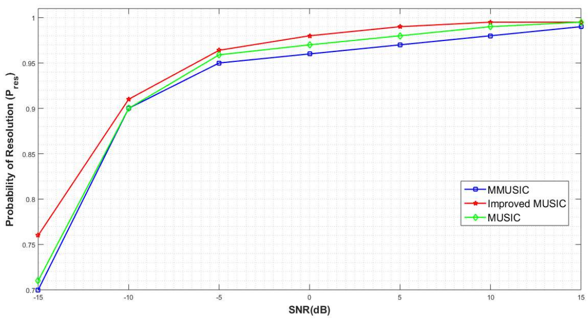

Fig. 7. Variation of P_{res} of DOA estimation with SNR for standard MUSIC, MMUSIC and improved MUSIC algorithms

V. CONCLUSION

In this paper, the method of standard MUSIC algorithm is modified by conjugate transformation, performed prior to eigenvalue decomposition. The improved method achieves better results than the standard counterpart in resolving near correlated sources. Simulation outcomes show that the improved MUSIC algorithm outclasses both standard and modified MUSIC methods in terms of root-mean-square-error (RMSE) at low SNR values. The proposed improved MUSIC algorithm can find suitable applications in radar and sonar tracking applications in resolving near correlated sources.

Conflict of Interest: The authors declare no conflict of interest.

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